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# UTILITY PATENT APPLICATION TRANSMITTAL

(Only for new nonprovisional applications under 37 C.F.R. § 1.53(b))

Attorney Docket No.	KAPUR 5-10
First Inventor or Application Identifier	Sharad Kapur
Title	SYSTEM AND METHOD FOR DETERMINING CAPACITANCE FOR LARGE-SCALE INTEGRATED CIRCUITS
Express Mail Label No.	EL053867968US

## APPLICATION ELEMENTS

See MPEP chapter 600 concerning utility patent application contents.

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- ☒ \* Fee Transmittal Form (e.g., PTO/SB/17)  
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(preferred arrangement set forth below)
  - Descriptive title of the Invention
  - Cross References to Related Applications
  - Statement Regarding Fed sponsored R & D
  - Reference to Microfiche Appendix
  - Background of the Invention
  - Brief Summary of the Invention
  - Brief Description of the Drawings (if filed)
  - Detailed Description
  - Claim(s)
  - Abstract of the Disclosure
- ☒ Drawing(s) (35 U.S.C. 113) [Total Sheets **4**]
- Oath or Declaration [Total Pages **1**]
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(for continuation/divisional with Box 16 completed)
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Signed statement attached deleting inventor(s) named in the prior application, see 37 C.F.R. §§ 1.63(d)(2) and 1.33(b).

- ☐ Microfiche Computer Program (Appendix)
- Nucleotide and/or Amino Acid Sequence Submission (if applicable, all necessary)
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## ACCOMPANYING APPLICATION PARTS

- ☐ Assignment Papers (cover sheet & document(s))
- ☐ 37 C.F.R. § 3.73(b) Statement of Power of Attorney (when there is an assignee)
- ☐ English Translation Document (if applicable)
- ☐ Information Disclosure Statement (IDS)/PTO-1449
- ☐ Preliminary Amendment
- ☐ Return Receipt Postcard (MPEP 503) (Should be specifically itemized)
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SYSTEM AND METHOD FOR DETERMINING CAPACITANCE  
FOR LARGE-SCALE INTEGRATED CIRCUITS

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# SYSTEM AND METHOD FOR DETERMINING CAPACITANCE FOR LARGE-SCALE INTEGRATED CIRCUITS

## TECHNICAL FIELD OF THE INVENTION

The present invention is directed, in general, to integrated  
5 circuit (IC) design and, more specifically, to a system and method  
for determining capacitance for large-scale ICs.

## BACKGROUND OF THE INVENTION

Interconnect analysis for modern ICs has traditionally  
required large scale capacitance calculations. However, such  
10 analysis required the geometry of the ICs to be described in  
detail, which resulted in millions, or even billions, of polygonal  
elements. The interconnect analysis yielded a sparse capacitance  
matrix of dominant coupling coefficients between conductors  
(commonly called "nets"). A net is formed of metallic pieces that  
15 are located within the IC and coupled together such that they  
cooperate to conduct current between two points. The net also has  
a capacitance associated with it.

Traditionally, capacitance calculations have not required  
great accuracy (accuracies of 10-20% were acceptable). To achieve

this relatively low accuracy, pattern matching schemes consisting of precharacterized libraries were used. However, nets such as clock nets, power grids, sensitive analog nodes or critical paths of a circuit require a higher accuracy in their solution. Unfortunately, as the requirement for accuracy increased, pattern matching solutions became insufficient.

Numerical approaches were then explored in an effort to increase the accuracy in capacitive calculations. Conventional numerical approaches rendered conductor surfaces (nets) in terms of discrete elements. Enforcing the equation either as a set of collocation points or as a distribution, as with a Galerkin scheme, led to a dense linear system. The large size of these systems usually required that they be solved with Krylov iterative methods. The required matrix-vector products were accelerated with compressed-matrix schemes (such as FastCap), pre-corrected Fast Fourier Transforms, or Singular Value Decomposition ("SVD")-based representations. While these methods are capable of high accuracy, they are restricted to systems of only a few hundred thousand unknowns. Even with only one unknown per polygon, large problems having millions of polygons remained well beyond the reach of these methods.

The industry further tried what is conventionally termed a "random walk" method, based on the well-known Monte Carlo method.

Unfortunately, the random walk method required a significant amount of central processing unit ("CPU") time to achieve high accuracy. For this reason, larger problems involving millions of polygons, proved to be unsatisfactory.

5       Next, finite-difference or finite-element methods were attempted. These methods required that a three-dimensional mesh be imposed on the geometry of the capacitive structure. However, these methods created more unknowns than did other numerical methods, and therefore were even less desirable.

10       Accordingly, what is needed in the art is way to increase the accuracy of IC capacitance calculations without incurring excessive computational time or difficulty.

## SUMMARY OF THE INVENTION

To address the above-discussed deficiencies of the prior art, the present invention provides, for use in an integral equation formulation of capacitance, a system for, and method of, generating  
5 a representation of charge distribution for a given capacitive structure (which may be an integrated circuit). In one embodiment, the system includes: (1) a charge variation function generator that creates a multidimensional charge variation function that is independent of a conductive geometry of the structure and (2) a  
10 conductive geometry generator, associated with the charge variation generator, that creates a conductive geometry that is independent of charge variation in the structure, the charge variation function and the conductive geometry employable in the integral equation formulation to reduce a complexity thereof.

15 The present invention therefore introduces the broad concept of rendering the charge variation and conductive geometry independent of one another, thereby significantly reducing the complexity of the subsequent integral equation formulation required to determine a structure's capacitance.

20 In one embodiment of the present invention, the integral equation formulation is a Fast Distribution Method. Those skilled in the pertinent art will understand, however, that the broad scope

of the present invention is not limited to a particular integral equation formulation.

In one embodiment of the present invention, the charge variation function is a three-dimensional function. Three-dimensional functions are particularly useful in describing integrated circuit structures. However, the present invention is not limited to a particular dimensionality.

In one embodiment of the present invention, the charge variation function is a smooth function of spatial location. Smooth functions can be accurately represented by low-order polynomials and are therefore advantageous.

In an illustrated embodiment of the present invention, the conductive geometry generator iteratively creates the conductive geometry. In the illustrated embodiment of the present invention, the charge variation function generator employs a generalized minimal residual-based krylov method to determine the multidimensional charge variation function.

In one embodiment of the present invention, the conductive geometry is represented in an octtree. The structure and function of octtrees will be set forth in the Detailed Description that follows. Those skilled in the pertinent art will understand, however, that octtrees are not required by the present invention.

The foregoing has outlined, rather broadly, preferred and

alternative features of the present invention so that those skilled in the art may better understand the detailed description of the invention that follows. Additional features of the invention will be described hereinafter that form the subject of the claims of the invention. Those skilled in the art should appreciate that they can readily use the disclosed conception and specific embodiment as a basis for designing or modifying other structures for carrying out the same purposes of the present invention. Those skilled in the art should also realize that such equivalent constructions do not depart from the spirit and scope of the invention in its broadest form.



## BRIEF DESCRIPTION OF THE DRAWINGS

For a more complete understanding of the present invention, reference is now made to the following descriptions taken in conjunction with the accompanying drawings, in which:

5       FIGURE 1 illustrates a capacitive structure that forms one environment with which the present invention can operate on to reduce the complexity thereof;

10       FIGURE 2 illustrates a block diagram a system for determining a capacitance of an integrated circuit constructed according to the principles of the present invention;

15       FIGURE 3A illustrates a method of refining the approximation of the geometry of a capacitive structure using octtrees;

      FIGURE 3B illustrates the refined octtree representation of the geometry of a capacitive structure of FIGURE 3A; and

      FIGURE 4 illustrates a table of the time and memory required to determine the capacitance of a section of a digital chip employing an embodiment of an algorithm of the present invention.

## DETAILED DESCRIPTION

Referring initially to FIGURE 1, illustrated is a capacitive structure that forms one environment with which the present invention can operate to reduce the complexity thereof. The capacitive structure 100 illustrates an example of three nets C1, C2, C3 in an integrated circuit (IC). A net is comprised of metallic pieces connected together to allow a current to pass from a first point in an IC to a second point in the IC. The net has an associated capacitance. The net may pass through and include portions of different IC structures that can affect the associated capacitance of the net. The IC structures may include traces, capacitors, switches, resistors and other elements that make up an IC.

The net C1 is a block representation of a first net within the capacitive structure 100. The net C2 is a block representation of a second net within the capacitive structure 100. The net C2 is associated with, but not coupled to, the net C1. The net C3 is a block representation of a third path within the capacitive structure 100 and is associated with, but not coupled to either one of, the net C1 and the net C2.

The capacitive value of the capacitive structure 100 depends upon the geometry of each of the nets C1, C2, C3, the capacitance

for each of the nets C1, C2, C3 and the electromagnetic interplay between the nets C1, C2, C3. The capacitive value also depends upon the charge distribution for the capacitive structure 100.

As previously described, the difficulty with previous capacitance calculation approaches is that they require the geometry of the nets to be captured accurately, causing the minimum problem size for the capacitance calculation to exceed practical limits. The present invention addresses this substantial shortcoming by providing a method that does not require that the geometry of the nets be accurately captured.

Consider the capacitance of the net C1; it is affected by the net C3. However, the net C3 is located far enough away that the exact geometry of the net C3 is not critical. Therefore, were the net C3 to be replaced with a simpler geometric description, the number of unknowns could be reduced. In particular, the number of unknowns needed to determine the capacitance of the capacitive structure 100 would then be constrained by the charge distribution, rather than the geometry, of the nets.

In one embodiment of the present invention, the charge variation is decoupled from the geometry of the nets. Decoupled means the charge variation is determined separately from the geometry. However, the charge variation and the geometry are used in the actual capacitance calculation. Also, the surface charge

distribution is represented as a smooth function of spatial location which is projected onto the geometry. This results in the number of unknowns being dictated solely by the charge variation.

The geometry of the nets C1, C2, C3 is described hierarchically and captured to a level of detail needed to determine an accurate solution. For example, in FIGURE 1, the net C1 and the net C2 are captured with relatively detailed geometric descriptions. However, the net C3 is captured with relatively coarse geometric descriptions.

With the charge variation decoupled from the geometry of the nets C1, C2, C3, the capacitance is calculated using an iterative linear solution. In one embodiment of the present invention, the present invention uses a Krylov method based on a Generalized Minimal Residual (GMRES) method to solve the capacitance of the nets C1, C2, C3. For more background material on this subject, see GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems, by Y. Saad and M. H. Shultz, SIAM Journal on Scientific and Statistical Computing, 7(3):856-69, 1986 (incorporated herein by reference).

Unlike standard GMRES, the present invention represents the net geometry as a set of the charge distribution functions rather than a set of discrete vectors. This method of representation is founded on the assumption that the charge variation function is a

smooth function of spatial location. With the advantageous use of the charge distribution functions, the present invention makes it possible to solve problems that are intractable with conventional methods.

5           Turning now to FIGURE 2, illustrated is a block diagram a system for determining a capacitance of an IC, generally designated 200, constructed according to the principles of the present invention. The system 200 comprises an integral equation formulator 210, a charge variation function generator 220, and a  
10       conductive geometry generator 230. The integral equation formulator 210 determines the total capacitance of all the nets C1, C2, C3 within the IC.

15           In determining the capacitance for the nets C1, C2, C3, the integral equation formulator 210 uses an iterative linear solution method. The iterative linear solution method is a recursive method that starts with a first guess of the geometry of a net and converges toward a charge distribution function that is within acceptable limits when applied to the geometry. If the charge distribution function is not within acceptable limits, the geometry  
20       is divided into smaller geographic elements called subdivisions. The iterative linear solution method is then performed on the subdivided structure. The resolution of the charge distribution function and the subdivision of the geometry continues until the

charge distribution function for each of the subdivisions is within acceptable limits.

Once the integral equation formulator 210 has found the solutions for the charge distribution functions for each of the nets C1, C2, C3, the integral equation formulator 210 determines the capacitance for the nets C1, C2, C3.

Associated with the integral equation formulator 210 is the charge variation function generator 220. The charge variation function generator 220 creates charge variation functions that the integral equation formulator 210 uses to refine the representation of the charge distribution. For each iteration of linear solution method, the charge variation function generator 220 creates a charge variation function for a particular subdivision of the geometry.

Also associated with the integral equation formulator 210 is the conductive geometry generator 230. The conductive geometry generator 230 generates an advantageous description of the geometry of the IC by performing an adaptive subdivision of the nets C1, C2, C3. Over the course of the iterative linear solution method, the conductive geometry generator 230 may further subdivide the existing subdivisions of the geometry to assist the integral equation formulator 210 in reaching an acceptable solution.

In the illustrated embodiment, the integral equation

formulator 210 uses a two-component representation of the charge distribution. The first component,  $\chi_R$ , is the characteristic function of the surfaces of the net and represents the geometry, without regard to the charge distribution. The second component is  
5 a charge distribution function  $f$  that represents the charge distribution, without regard to the geometrical layout. Since the charge distribution and the geometry are wholly decoupled, the representation of the charge distribution projected on the geometry requires far fewer unknowns than previous methods.

10 In previous numerical analysis methods, the geometry of the entire set of net surfaces were incorporated into the calculations for any subdivision of the geometry. As a result, these methods required solving for a relatively large number of unknowns to account for all of the geometry of the nets C1, C2, C3. This  
15 included all the geometry that was far-removed from the subdivision being considered. In contrast, the present invention advantageously does not require the incorporation of all the far-removed subdivisions. This greatly reduces the number of unknowns required in solving for capacitance of the IC.

20 In one embodiment of the present invention, the characteristic geometry function for the set of net surfaces,  $\chi_R$ , is a predefined function. In an alternate embodiment of the present invention,  $\chi_R$  is calculated from information related to the capacitive structure

of the IC.  $\chi_R$  is scalar-valued function that has a n-dimensional position as the input. To illustrate the form of  $\chi_R$ , suppose the geometry to be described is the unit square in the xy-plane. The form of  $\chi_R$  for this geometry then would be:

$$\chi_R = s(x)s(y)\delta(z),$$

where  $s(x)=1$  for  $0 \leq x \leq 1$  and  $s(x)$  is zero elsewhere, and  $\delta(z)$  is the standard Dirac delta function. Those skilled in the art are familiar with the properties and use of Dirac delta functions.

The function  $f$ , which represents the charge distribution without regard to geometry, is evaluated over  $R$ , the set of surfaces of the nets C1, C2, C3. The surface integrals over  $R$  are reformulated as volume integrals via the relation:

$$\int_R f = \int_V f \chi_R$$

where  $V$  is the volume that the net comprises.

To solve the iterative linear solution, the integral equation formulator 210 uses an initial guess  $g$ , for the charge distribution, and a starting subdivision of the geometry of the net. The integral equation formulator 210 also creates the projection matrix  $P_R$  from  $\chi_R$ ,

$$P_R = \sum_k \langle \chi_R, l_k \rangle P_k$$



The bracketed expression above is the standard inner product for functions:

$$\langle f, g \rangle = \int_V fg$$

Also in this equation,  $P_k$  is a primitive projection matrix where the (i,j) entry of  $P_k = \int_V l_i l_j l_k$ . Finally the  $l_i$ s represent the choice of orthogonal polynomials. In one embodiment of the present invention, Legendre polynomials form the system of orthogonal polynomials. One skilled in the pertinent art is familiar with and the use of Legendre polynomials. Also,  $P_R^b$  will represent that part of  $P_R$  that contains information of the geometry of subdivision b.

Using the initial guess  $g$  for the charge distribution and the starting subdivision of the geometry of the net, the integral equation formulator 210 determines charge distributions for a given potential on the nets C1, C2, C3. From the charge distributions, the integral equation formulator 210 determines the capacitance of the nets C1, C2, C3.

In the determination of the charge distributions, the iterative linear solution uses the following fundamental equation that relates the charge and the potential:

$$\psi(r) = \int_R G(r, r') \rho(r') dr'$$

where  $G(r, r')$  is the Green's function for the geometry of the IC and  $\rho(r')$  is the charge density. Those skilled in the art are familiar with the properties and use of Green's functions.

In one embodiment of the present invention, the fundamental equation relating the charge and the potential is expressed as:

$$\psi = MP_R f$$

where the  $\psi$  is a potential distribution and  $f$  represents the charge distribution without regard to the geometry. From this equation, the integral equation formulator 210 can determine the charge distribution function  $f$ .

In one embodiment of the present invention, the integral equation formulator 210 uses a Krylov method based on a variation of a Generalized Minimal Residual Method (GMRES) for the iterative linear solution. Background information concerning GMRES is discussed in GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems, by Y. Saad and M. H. Shultz, SIAM Journal on Scientific and Statistical Computing, 7(3):856-69, 1986 (incorporated herein by reference).

GMRES is used to solve for the charge distribution function  $f$  in the following equation:

$$\|MP_R f - \psi_0\|_R = 0$$

or, equivalently:

$$P_R M P_R f = P_R \psi_0,$$

where  $\psi_0$  is a potential which is 1 over an individual net and 0 elsewhere. Those skilled in the pertinent art are familiar with the application of GMRES. Nevertheless, below is a description of the process both to give a complete picture of the iterative linear solution process and to show the advantages of decoupling the charge variation from the geometry.

The integral equation formulator 210 starts iterative linear solution with an initial guess  $g$  for the charge distribution and an initial geometry. The integral equation formulator 210 also determines  $\chi_R^b$ , which represents the geometry of the box  $b$  in the subdivision, and  $P_R^b$ , which is the part of the projection matrix  $P_R$  which covers the box  $b$  (see FIGURE 3A for more information concerning geometry subdivisions and boxes). From this information, the integral equation formulator 210 computes a potential  $\psi$  based upon the initial charge distribution guess  $g$  and the current subdivision.

The integral equation formulator 210 also determines the desired potential  $\psi_0$ :

$$P_R \psi_0 = P_S o$$

where  $o$  is the constant function 1 and  $P_S$  is defined as  $P_R$  is over

an individual net S. The integral equation formulator 210 then determines the difference between the desired potential  $\psi_0$  and the potential  $\psi$ . This difference,  $\psi_0 - \psi$ , becomes first charge variation function  $f_1$ . Note that  $f_1$  is not a representation of the charge distribution described by  $f$ .  $f_1$  is a function used to modify the charge distribution function  $f$  to reach a desired resolution.

The integral equation formulator 210 determines the ratio  $\gamma/\beta$ , where  $\gamma = \|f_1\|_R$  and  $\beta = \|\psi_0\|_R$ . If the ratio is sufficiently small, the iterative linear solution is complete. If the ratio is unacceptably large, the integral equation formulator 210 normalizes  $f_1$  and proceeds with the linear iterative solution.

The integral equation formulator 210 then invokes the charge variation function generator 220 to create a charge variation function which refines the description of the charge distribution. At the beginning of the iterative process the charge variation function generator 220 creates a second charge variation function  $f_2$ . In general, the formula for the creation of the  $j+1^{\text{th}}$  charge variation function from the  $j^{\text{th}}$  charge variation function is:

$$f_{j+1} = MP_R^b f_j .$$

Where  $M$  is an operator that converts a charge density to a potential distribution. The  $(i,j)$  entry of the matrix representation of  $M$  is:

$$M^{bc} = \int_b \int_c l_i^b l_j^c G(r, r') dr' dr$$

Where  $l_i^b$  is the  $i^{\text{th}}$  moment in box  $b$ . Note that the operation of  $M$  involves all boxes, not just box  $b$ .  $G(r, r')$  is the Green's function for the set of net surfaces  $R$ . Those skilled in the art  
 5 are familiar with the properties and use of Green's functions.

The charge variation function generator performs the operation  $M$  using a variant of the Fast Multipole Method (FMM) algorithm called the Fast Distributed Method (FDM). The FDM differs from the FMM in several areas. First, the FDM omits the FMM's initial step  
 10 of computing multipole representations from point charges. FDM omits this step because the input is already in terms of a charge distribution. Second, the FDM omits FMM's direct point-to-point interactions. Instead, the FDM uses a source charge distribution in a cube. The potential distributions are calculated in all  
 15 neighboring cubes, including the source cube itself.

Third, the FDM omits FMM's final step of interpolating point potentials from the local expansions. The FDM omits the last step, because the desired output is a potential distribution. Finally, FDM uses Legendre polynomial expansions for both charge  
 20 distributions and local expansions. The FMM uses multipole expansions for representing charge distributions and local expansions for representing potential distributions.

The FMM algorithm, Legendre polynomial expansions and their use are well known to those skilled in the pertinent art. Background information concerning numerical analysis and capacitance calculations is discussed in Introduction to Numerical Analysis, by J. Soer and R. Bulirsh, Springer-Verlag 1979 and in Preconditioned, Adaptive, Multipole-Accelerated Iterative Methods for Three-Dimensional First-Kind Integral Equations of Potential Theory, by K. Nabors, et al., SIAM Journal on Scientific and Statistical Computing, 15(3):713-735, May 1994 (both incorporated herein by reference).

The use of the FDM results in a significant savings in time over both the analytical calculation using the explicit form of M and the FMM. The savings are due to the omission of several FMM steps and the omission of FMM's direct point-to-point interactions.

Once the charge variation function generator 220 has created the charge variation function  $f_{j+1}$ , the integral equation formulator 210 uses this new function to create the (i, j) entry of a Hessenberg matrix:

$$H_{ij} = \left\langle f_i, f_{j+1} \right\rangle_R$$

for all  $i \leq j$ .

The integral equation formulator 210 uses the Hessenberg matrix to solve for the new coefficients of the charge variation functions to

be used to further refine the charge distribution function  $f$ .

After the entries in the Hessenberg matrix are computed, the integral equation formulator 210 then orthogonalizes  $f_{j+1}$  from the other  $f_i$ s. The charge variation function generator 220 then  
5 creates the  $(j+1,j)$  entry of the Hessenberg matrix by the expression:

$$H_{j+1,j} = \|f_{j+1}\|_R.$$

This expression is the standard norm of functions defined by:

$$\|f\|_R = \sqrt{\langle f, f \rangle} = \sqrt{\int_R f^2}$$

10 After the creation of this last entry of the Hessenberg matrix, the integral equation formulator 210 then normalizes  $f_{j+1}$ . The orthogonalization and the normalization of  $f_{j+1}$  create orthonormal charge variation functions. As a result, the integral equation formulator 210 is able to obtain the correct charge distribution  
15 function  $f$  very quickly.

The integral equation formulator 210 then solves for  $\mathbf{x}$ , the coefficient vector for the charge variations, using least-squares on the equation:

$$H\mathbf{x} = \gamma \mathbf{e}_1$$

where H is the Hessenberg matrix generated by the integral equation  
 formulator 210 as explained above.  $\mathbf{e}_1$  is the 1<sup>st</sup> unit vector with  
 the form (1,0,0,0,0, . . .) (with j number of zeros). The vector  $\mathbf{x}$   
 is the coefficient vector for the charge distributions and  $\gamma$  is the  
 5 norm of the first charge variation function.  $\gamma$  is defined as:

$$\gamma = \|\mathbf{f}_1\|_R.$$

Since the charge variation and the geometry are decoupled, the  
 representation of the  $\mathbf{f}_i$  will not become very large as compared to  
 the previous methods. Using the residual of the least squares, the  
 10 integral equation formulator 210 computes the factor r as the 2-  
 norm of the least squares residual.

The integral equation formulator 210 then generates a new  
 guess for the charge distribution function f using the equation:

$$f = g + \sum_{i=1}^j x_i f_i$$

15 The integral equation formulator 210 then determines the ratio  
 $r/\beta$ . Where r is the 2-norm of the least squares residual r and  $\beta$   
 is the norm of the desired potential (defined earlier as  $\beta = \|\psi_0\|_R$ ).  
 If the ratio is within acceptable limits, the charge distribution  
 has converged and the integral equation formulator 210 terminates  
 20 the iterative linear solution.



If the ratio is not within acceptable limits, the integral equation formulator 210 uses the new guess for the charge distribution function  $f$ , calculated above, as the starting point for the next iteration of the iterative linear solution. However, before the integral equation formulator 210 performs the next iteration, the integral equation formulator 210 must determine if the current subdivision requires a more refined approximation.

The integral equation formulator 210 determines if the current subdivision  $b$  requires refinement by calculating a charge-geometry error  $e$ . In one embodiment of the present invention, the integral equation formulator calculates the charge-geometry error  $e^b$  associated with box  $b$  in the subdivision using the following equation:

$$e^b = \sum_{i=k}^{N-1} f_i^b l_i$$

Where  $l_i$  is the  $i^{\text{th}}$  moment basis function.  $f$  represents the distribution of the charge without any reference to the geometry of the IC and  $f_i^b$  is the component of  $f$  along the  $i^{\text{th}}$  moment for the charge function contained in the subdivision  $b$ . The summation of  $l_i$  between  $k$  and  $N-1$  represent the high order polynomials. Finally,  $N$  is the number of moments used in the expansion of  $f$ .

The integral equation formulator 210 then determines if the

charge-geometry error is within acceptable limits using a charge-geometry error criterion. The charge-geometry error criterion is defined as:

$$\|e^b\|_R < \frac{\varepsilon}{\|f\|_S}$$

5 Where S is the minimal subdivision containing the selected net.  $\varepsilon$  is a small constant that determines the allowable tolerance of the criterion. Also,  $\|e^b\|_R$  is the magnitude of the charge-geometry error in subdivision b. Note that the error threshold is relative to the charge distribution f only on the subdivision of the selected net.

10 If the charge-geometry error criterion is within acceptable limits, the integral equation formulator 210 has finished the resolution of the charge variation function. If the charge-geometry error criterion is not within acceptable limits, then the  
15 integral equation formulator 210 calls the conductive geometry generator 230 to refine the approximation of the geometry by subdividing the current subdivision b. See FIGURE 3 for a discussion of the refining the approximation of the geometry of a net.

20 After the conductive geometry generator 230 has subdivided the

geometry, the integral equation formulator 210 then starts the iterative linear solution again on the new subdivision. The integral equation formulator 210 uses the last calculated charge distribution function  $f$  as the starting point for the calculations on the new subdivision.

Once the integral equation formulator 210 has completed the iterative linear solution calculations for all the nets, the integral equation formulator 210 then calculates the capacitance of the IC. To determine the capacitance, the total charge on each net is computed from the charge distribution. The charge on the net whose surface is  $S$  is  $\int_S f$ . The present invention is not limited to a single method of determining capacitance. Other methods of determining capacitance from charges on nets are within the broad scope of this invention. Background information concerning the calculation of the capacitance is discussed in Classical Electrodynamics, by John Jackson, John Wiley & Sons (3d edition, 1998) (incorporated herein by reference).

Turning now to FIGURE 3A, illustrated is a method of refining the approximation of the geometry of a capacitive structure (or net) using octtrees. An octtree is a generalized way of subdividing the geometry into rectangular "boxes" that are congruent within every level of subdivision. Each box in a subdivision is called a "leaf."

The conductive geometry generator 230 starts with the initial geometry 310 of a net. When the integral equation formulator 210 determines that the initial geometry 310 needs to be refined, the conductive geometry generator 230 divides the initial geometry 310 into three leaves shown in the first subdivision 320. Usually the geometry generator 230 will divide a given portion of a geometry into four equal leaves. However, the conductive geometry generator 230 will discard any leaf that is not required for the approximation of the geometry. In this instance, the conductive geometry generator 230 discarded the lower right leaf in first subdivision 320.

On the next iteration of refining the approximation of the geometry, the conductive geometry generator 230 subdivides the lower left leaf, of the first subdivision 320, into three smaller leaves shown in the second subdivision 330. In this iteration, the conductive geometry generator 230 discarded the upper right leaf of the second subdivision 330.

On the final iteration of refining the approximation, the conductive geometry generator 230 subdivides the upper right leaf, of the first subdivision 320, into three smaller leaves shown in the third subdivision 340. In this iteration, the conductive geometry generator 230 discarded the lower right leaf of the third subdivision.

Now turning to FIGURE 3B, illustrated is the refined octtree approximation of the geometry of the capacitive structure of FIGURE 3A. The refined approximation 350 shows the results from the conductive geometry generator subdividing the initial conductive geometry 310. Note that the leaves are not of equal size. In particular, the leaf on the upper-left quadrant of 350 is four times larger than the other leaves illustrated. This is a result of the fact that although the n-dimensional boxes at a particular level are all the same size. However, all the leaves are not required to be the same size since the leaves are not generated from the same level of subdivision. The subdivision of one leaf does not mandate the subdivision on all other leaves that are the same size.

There are various advantageous reasons for using octtrees to approximate the geometry of a conductive structure. First, the description of any n-dimensional space is relatively simple. Second, octtrees provide a simple description of the geometry. Third, the boxes (leaves) that comprise an octtree fit together in a natural way regardless of how often they are subdivided. Fourth, it is a relatively simple task to ignore sections that are not important to the approximation of the geometry. Finally, any octtree can be refined to an arbitrary level of accuracy.

Turning now to FIGURE 4, illustrated is a table 400 of the

time and memory required to determine the capacitance of a section of a digital chip employing an embodiment of an algorithm of the present invention. The algorithm was performed on an ULTRASparc II workstation (manufactured by Sun Microsystems) having a 400  
5 Megahertz (MHZ) CPU and 1 Gigabyte of memory, which is a typical system that may be employed in performing the capacitance calculations of the present invention. The algorithm used octtrees with square subdivisions as the geometrical structure and employed a Krylov algorithm to solve for the charge variation. The  
10 tolerance of the system was set at about 5%.

The time and memory requirements illustrated are derived from calculations of an interconnect for the section of the digital chip. The section is about 0.5 mm on a side and is fabricated using 0.25 micron complementary metal oxide semiconductor ("CMOS")  
15 technology. The section contains 3,400 nets and geometry is described with 600,000 rectangles. Solving for the capacitance of a critical net required approximately two hours of CPU time and 610 Megabytes of memory. The calculation required a large amount of memory, in part, because this critical net of the digital chip is  
20 very close to many other nets.

The table 400 illustrates an exemplary time required by each part of the algorithm at various levels of refinement. The table 400 includes columns for (1) the number of cubes, (2) the time per

iteration for the Fast Distribution Method, (3) the geometry manipulation time, including miscellaneous operations, (4) the total time per iteration and (5) the memory used. At the finest level there were approximately 5.8 million unknowns (with 35 moments for each cube). The algorithm required about 100 seconds to perform a FDM calculation.

The present invention thus allows companies to solve problems many times larger than possible using other conventional methods. The companies may therefore perform computational tests of proposed ICs without having to construct the proposed IC. The present invention provides the end user with the benefits of flexibility and speed. Flexibility may result from a choice of methods for solving for both the charge variation and the advantageous geometry. Speed is provided by the categorical division of the problem between solving for the charge variation and solving for the advantageous geometry.

Although the present invention has been described in detail, those skilled in the art should understand that they can make various changes, substitutions and alterations herein without departing from the spirit and scope of the invention in its broadest form.

WHAT IS CLAIMED IS:

1. For use in an integral equation formulation of capacitance, a system for generating a representation of charge distribution for a given capacitive structure, comprising:

a charge variation function generator that creates a multidimensional charge variation function that is independent of a conductive geometry of said structure; and

a conductive geometry generator, associated with said charge variation generator, that creates a conductive geometry that is independent of charge variation in said structure, said charge variation function and said conductive geometry employable in said integral equation formulation to reduce a complexity thereof.

2. The system as recited in Claim 1 wherein said integral equation formulation is a Fast Distribution Method.

3. The system as recited in Claim 1 wherein said charge variation function is a three-dimensional function.

4. The system as recited in Claim 1 wherein said charge variation function is a smooth function of spatial location.



5. The system as recited in Claim 1 wherein said conductive  
2 geometry generator iteratively creates said conductive geometry.

6. The system as recited in Claim 1 wherein said charge  
2 variation function generator employs a generalized minimal  
3 residual-based Krylov method to determine said multidimensional  
4 charge variation function.

7. The system as recited in Claim 1 wherein said conductive  
2 geometry is represented in an octtree.

8. For use in an integral equation formulation of capacitance, a method of generating a representation of charge distribution for a given capacitive structure, comprising:

creating a multidimensional charge variation function that is independent of a conductive geometry of said structure; and

creating a conductive geometry that is independent of charge variation in said structure, said charge variation function and said conductive geometry employable in said integral equation formulation to reduce a complexity thereof.

9. The method as recited in Claim 8 wherein said integral equation formulation is a Fast Distribution Method.

10. The method as recited in Claim 8 wherein said charge variation function is a three-dimensional function.

11. The method as recited in Claim 8 wherein said charge variation function is a smooth function of spatial location.

12. The method as recited in Claim 8 wherein said creating said conductive geometry comprises iteratively creating said conductive geometry.

13. The method as recited in Claim 8 wherein said creating  
2 said multidimensional charge variation function comprises employing  
3 a generalized minimal residual-based Krylov method to determine  
4 said multidimensional charge variation function.

14. The method as recited in Claim 8 wherein said conductive  
2 geometry is represented in an octtree.

15. A system for determining a capacitance of a given  
integrated circuit, comprising:

a charge variation function generator that creates a  
multidimensional charge variation function that is independent of  
a conductive geometry of said integrated circuit;

a conductive geometry generator that creates a conductive  
geometry that is independent of charge variation in said integrated  
circuit; and

an integral equation formulator, associated with said charge  
variation generator and conductive geometry generator, that  
determines said capacitance of said integrated circuit based on  
said charge variation function and said conductive geometry.

16. The system as recited in Claim 15 wherein said integral  
equation formulator employs a Fast Distribution Method.

17. The system as recited in Claim 15 wherein said charge  
variation function is a three-dimensional function.

18. The system as recited in Claim 15 wherein said charge  
variation function is a smooth function of spatial location.

19. The system as recited in Claim 15 wherein said conductive

2 geometry generator iteratively creates said conductive geometry.

20. The system as recited in Claim 15 wherein said charge  
2 variation function generator employs a generalized minimal  
3 residual-based Krylov method to determine said multidimensional  
4 charge variation function.

21. The system as recited in Claim 15 wherein said conductive  
2 geometry is represented in an octtree.

**SYSTEM AND METHOD FOR DETERMINING CAPACITANCE  
FOR LARGE-SCALE INTEGRATED CIRCUITS**

**ABSTRACT OF THE DISCLOSURE**

For use in an integral equation formulation of capacitance, a  
5 system for, and method of, generating a representation of charge  
distribution for a given capacitive structure (which may be an  
integrated circuit). In one embodiment, the system includes: (1)  
10 a charge variation function generator that creates a  
multidimensional charge variation function that is independent of  
a conductive geometry of the structure and (2) a conductive  
geometry generator, associated with the charge variation generator,  
that creates a conductive geometry that is independent of charge  
variation in the structure, the charge variation function and the  
conductive geometry employable in the integral equation formulation  
15 to reduce a complexity thereof.

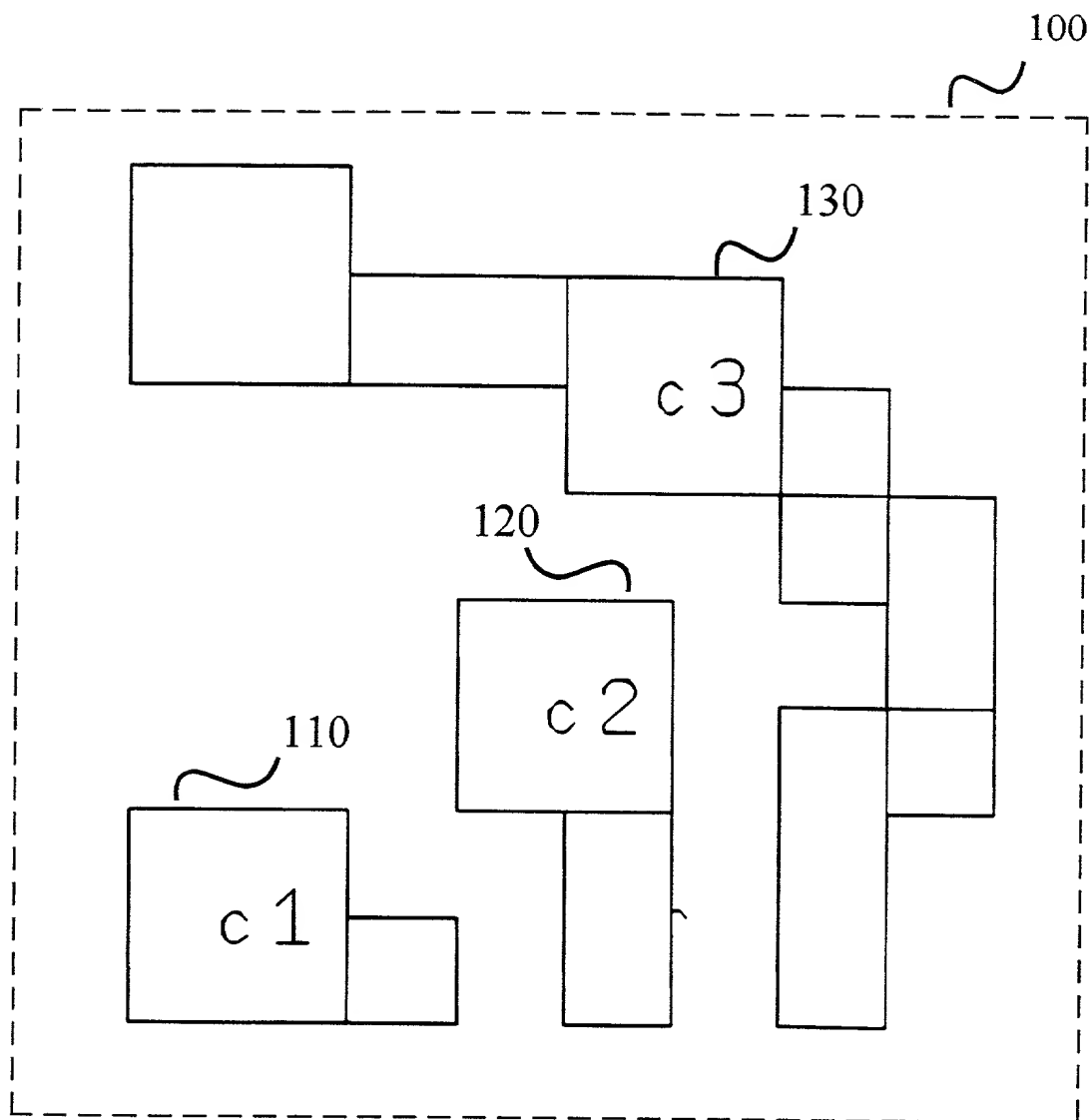


FIGURE 1

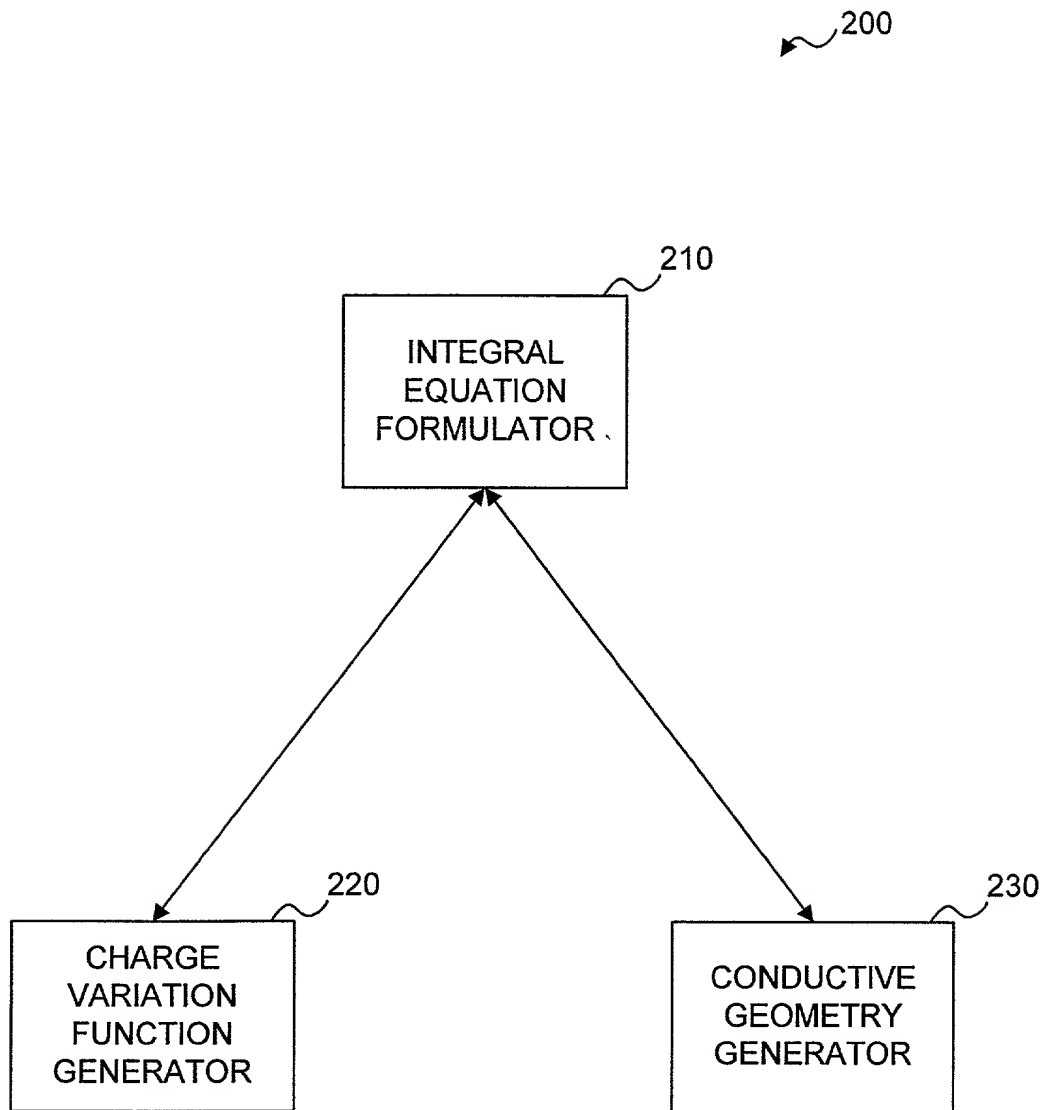


FIGURE 2



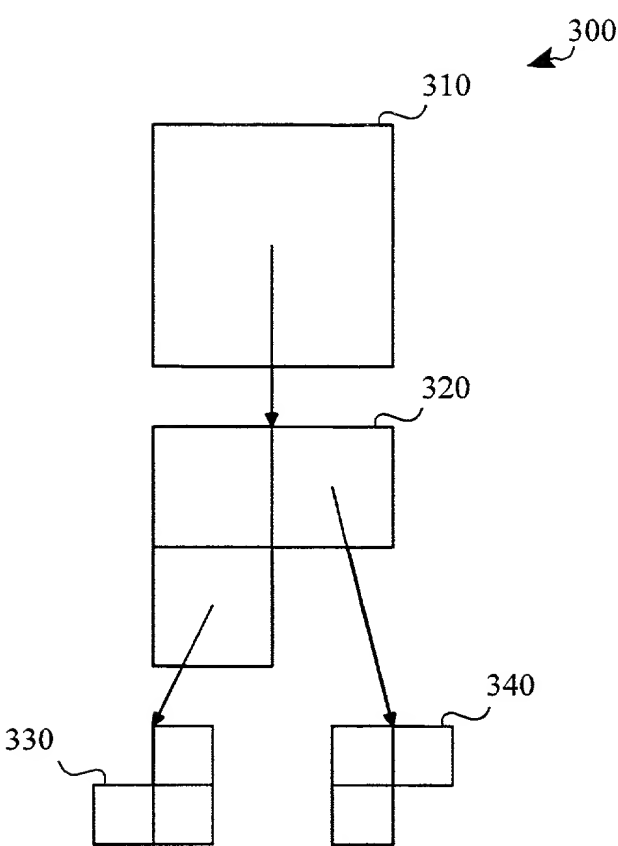


FIGURE 3A

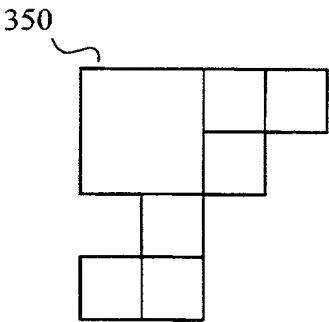


FIGURE 3B

Cubes	FDM time	Geometry time	Total time	Memory
3,100	1.3	0.4	2.4	30
7,700	3.5	0.9	5.9	42
13,500	6.8	1.6	11.3	59
33,500	17.7	3.6	29	130
78,000	44	7.2	64	280
167,000	100	15	142	610

**FIGURE 4**